

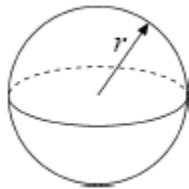
Year 12 Summer Task - Maths

Please complete the odd number questions. If you would like to do more than this please feel free, but these are a requirement.

For those students studying Further Maths, you will need to complete it all.

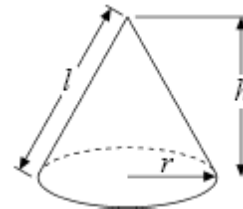
Volume of sphere $\frac{4}{3} \pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone $\frac{1}{3} \pi r^2 h$

Curved surface area of cone = $\pi r l$



1. (a) Write down the exact value of 3^{-2}

.....

(1)

(b) Simplify fully $\frac{7^2 \times 7^4}{7^3}$

.....

(2)

(c) Expand $(2 + \sqrt{3})(1 + \sqrt{3})$

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

.....

(2)

(Total 5 marks)

2. Work out

$$\frac{(5 + \sqrt{3})(5 - \sqrt{3})}{\sqrt{22}}$$

Give your answer in its simplest form.

.....

(Total 3 marks)

3. (a) Evaluate

(i) 3^{-2}

.....

(ii) $36^{\frac{1}{2}}$

.....

(iii) $27^{\frac{2}{3}}$

.....

(iv) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

.....

(5)

(b) (i) Rationalise the denominator of $\frac{21}{\sqrt{7}}$ and simplify your answer.

.....

(ii) Expand $(\sqrt{5} + 2\sqrt{3})(\sqrt{5} - 2\sqrt{3})$
Express your answer as simply as possible

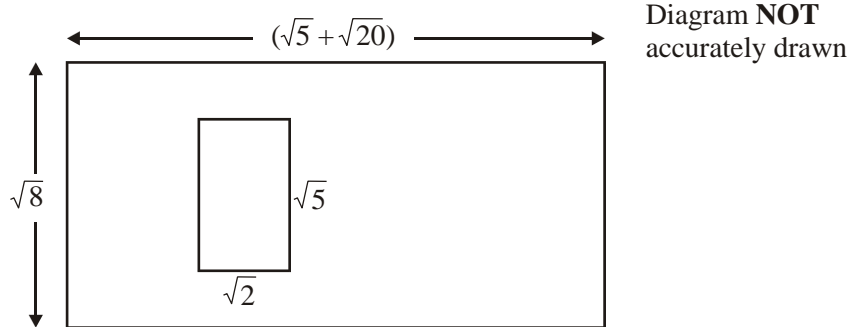
.....
(4)
(Total 9 marks)

4. (a) Find the value of $16^{\frac{1}{2}}$

.....
(1)

(b) Given that $\sqrt{40} = k\sqrt{10}$, find the value of k .

.....
(1)



A large rectangular piece of card is $(\sqrt{5} + \sqrt{20})$ cm long and $\sqrt{8}$ cm wide.

A small rectangle $\sqrt{2}$ cm long and $\sqrt{5}$ cm wide is cut out of the piece of card.

(c) Express the area of the card that is left as a percentage of the area of the large rectangle.

.....%
(4)
(Total 6 marks)

5. (a) Express $\frac{6}{\sqrt{2}}$ in the form $a\sqrt{b}$, where a and b are positive integers.

.....

(2)

The diagram shows a right-angled isosceles triangle.

The length of each of its equal sides is $\frac{6}{\sqrt{2}}$ cm.

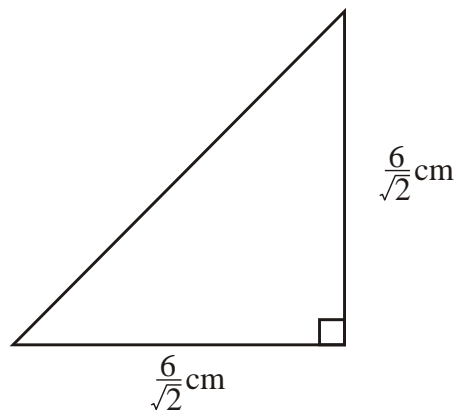


Diagram NOT accurately drawn

- (b) Find the area of the triangle.
Give your answer as an integer.

..... cm²

(2)

(Total 4 marks)

6. (a) Find the value of

(i) 64^0

.....

(ii) $64^{\frac{1}{2}}$

.....

(iii) $64^{-\frac{2}{3}}$

.....

(4)

(b) $3 \times \sqrt{27} = 3^n$

Find the value of n .

$n = \dots\dots\dots$

(2)

(Total 6 marks)

7. (a) Rationalise

$$\frac{1}{\sqrt{7}}$$

.....

(2)

(b) (i) Expand and simplify

$$(\sqrt{3} + \sqrt{15})^2$$

Give your answer in the form $n + m\sqrt{5}$, where n and m are integers.

.....

(ii)

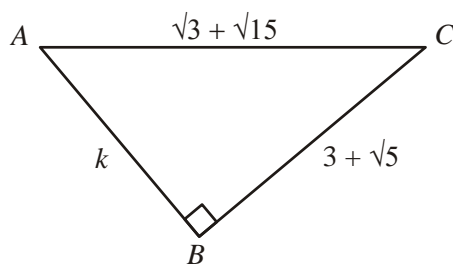


Diagram **NOT** accurately drawn

All measurements on the triangle are in centimetres.

ABC is a right-angled triangle.

k is a positive integer.

Find the value of k .

$k = \dots\dots\dots$

(5)

(Total 7 marks)

8. Work out the value of

(i) $(2^2)^3$

.....

(ii) $(\sqrt{3})^2$

.....

(iii) $\sqrt{2^4 \times 9}$

.....

(Total 4 marks)

9. Work out

(i) 4^0

.....

(ii) 4^{-2}

.....

(iii) $16^{\frac{3}{2}}$

.....

(Total 3 marks)

10. $2^x \times 2^y = 2^{10}$

and

$$2^x \div 2^y = 2^4$$

Work out the value of x and the value of y .

$$x = \dots\dots\dots$$

$$y = \dots\dots\dots$$

(Total 3 marks)

11. (a) Write down the value of $36^{\frac{1}{2}}$

.....

(1)

(b) $4n^{\frac{3}{2}} = 8^{\frac{1}{3}}$

Find the value of n .

$$n = \dots\dots\dots$$

(3)

(Total 4 marks)

12. Write down the value of

(a) 7^0

.....

(1)

(b) 4^{-1}

.....

(1)

(Total 2 marks)

13. Work out the value of

(i) $(2^2)^3$

.....

(ii) $(\sqrt{3})^2$

.....

(iii) $\sqrt{2^4 \times 9}$

.....

(Total 4 marks)

14. Simplify fully

(i) $(p^3)^3$

.....

(ii) $\frac{3q^4 \times 2q^5}{q^3}$

.....

(Total 3 marks)

15.

$$x = 2^p, y = 2^q$$

(a) Express in terms of x and/or y ,

(i) $2^p + q$

.....

(ii) 2^{2q}

.....

(iii) $2^p - 1$

.....

(3)

$$xy = 32$$

$$\text{and } 2xy^2 = 32$$

(b) Find the value of p and the value of q .

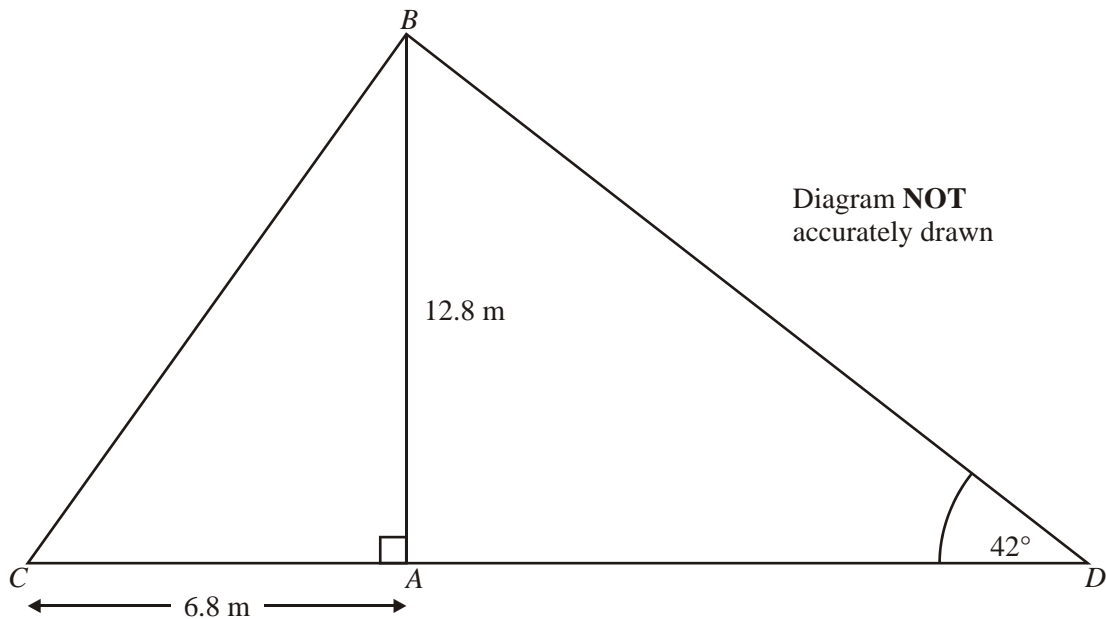
$$p = \dots\dots\dots$$

$$q = \dots\dots\dots$$

(2)

(Total 5 marks)

16. The diagram represents a vertical flagpole, AB .
The flagpole is supported by two ropes, BC and BD ,
fixed to the horizontal ground at C and at D .



$AB = 12.8$ m.
 $AC = 6.8$ m.
Angle $BDA = 42^\circ$.

- (a) Calculate the size of angle BCA .
Give your answer correct to 3 significant figures.

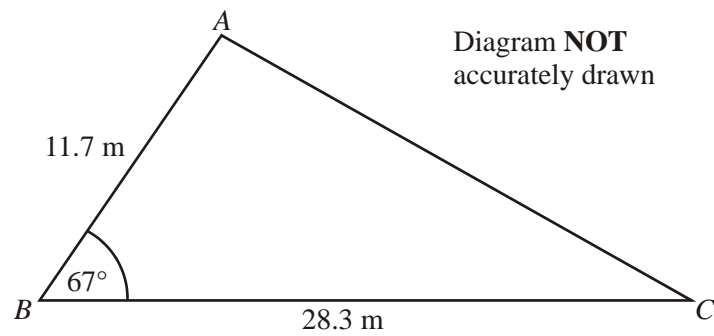
.....^o (3)

- (b) Calculate the length of the rope BD .
Give your answer correct to 3 significant figures.

..... m (3)

(Total 6 marks)

17.



$AB = 11.7 \text{ m.}$

$BC = 28.3 \text{ m.}$

$\text{Angle } ABC = 67^\circ.$

- (a) Calculate the area of the triangle ABC .
Give your answer correct to 3 significant figures.

..... m^2

(2)

- (b) Calculate the length of AC .
Give your answer correct to 3 significant figures.

..... m

(3)

(Total 5 marks)

18. The depth, D metres, of the water at the end of a jetty in the afternoon can be modelled by this formula

$$D = 5.5 + A \sin 30(t - k)^\circ$$

where

t hours is the number of hours after midday,
 A and k are constants.

Yesterday the low tide was at 3 p.m.
The depth of water at low tide was 3.5 m.

Find the value of A and k .

$A = \dots\dots\dots$

$k = \dots\dots\dots$

(Total 4 marks)

19.

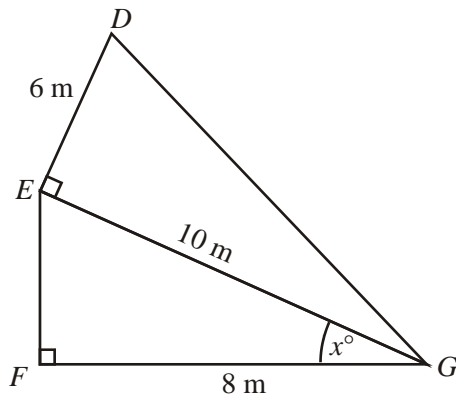


Diagram **NOT** accurately drawn

$DE = 6\text{ m.}$

$EG = 10\text{ m.}$

$FG = 8\text{ m.}$

Angle $DEG = 90^\circ$. Angle $EFG = 90^\circ$.

- (a) Calculate the length of DG .
Give your answer correct to 3 significant figures.

..... m

(3)

- (b) Calculate the size of the angle marked x° .
Give your answer correct to one decimal place.

.....^o

(3)

(Total 6 marks)

20.

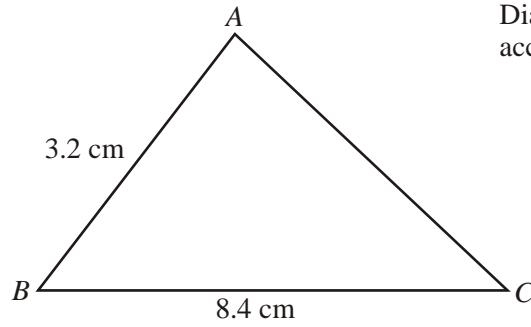


Diagram **NOT**
accurately drawn

$$AB = 3.2 \text{ cm}$$

$$BC = 8.4 \text{ cm}$$

The area of triangle ABC is 10 cm^2 .

Calculate the perimeter of triangle ABC .

Give your answer correct to three significant figures.

..... cm

(Total 6 marks)

21. Here is a right-angled triangle.

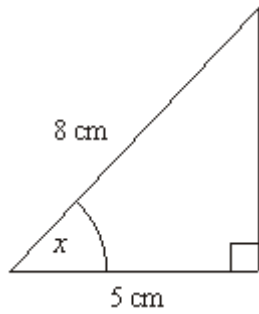


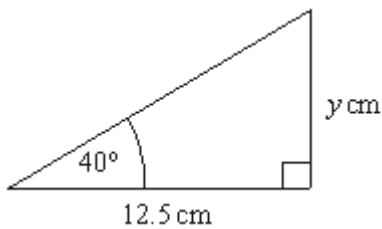
Diagram **NOT** accurately drawn

- (a) Calculate the size of the angle marked x .
Give your answer correct to 1 decimal place.

$$x = \text{.....}^\circ$$

(3)

Here is another right-angled triangle.



drawn

Diagram **NOT** accurately

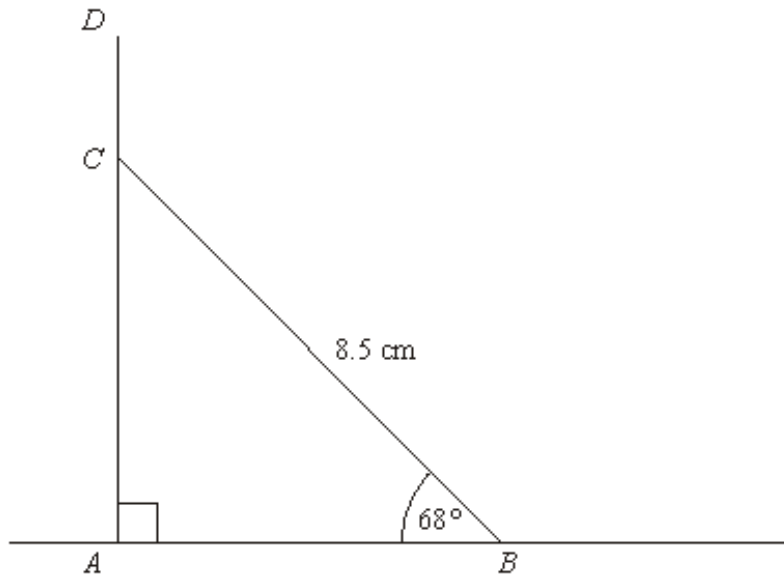
- (b) Calculate the value of y .
Give your answer correct to 1 decimal place.

$$y = \text{.....cm}$$

(3)

22.
accurately drawn

Diagram **NOT**



The diagram represents a vertical pole ACD .

AB is horizontal ground. BC is a wire of length 8.5 metres.

The height of the pole AD is 9 metres.

For the pole to be correctly installed, the length DC has to
be at least 1 metre.

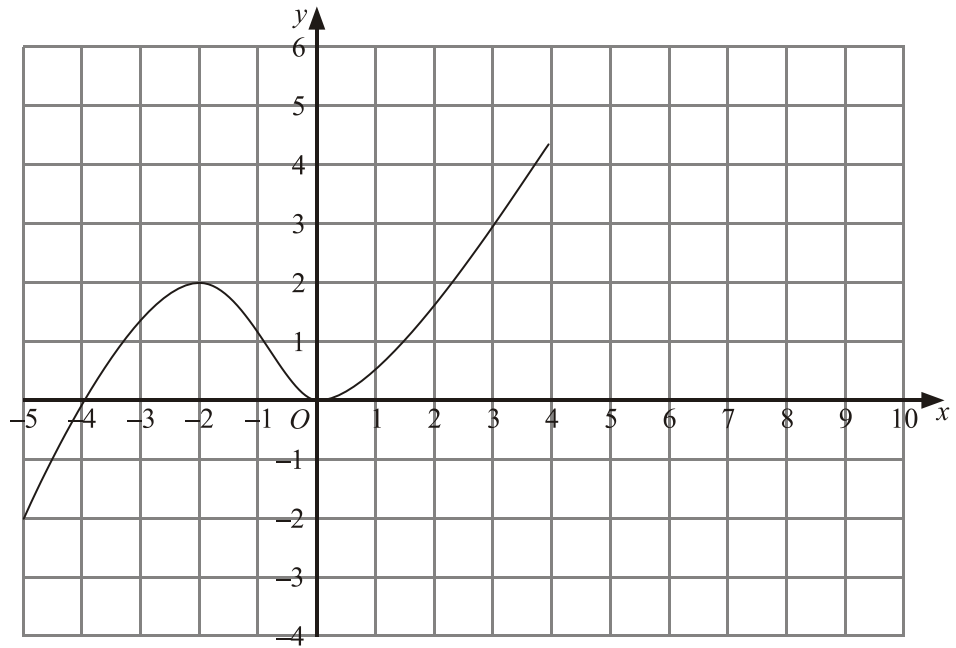
Show that the pole has been correctly installed.

.....

(Total 4 marks)

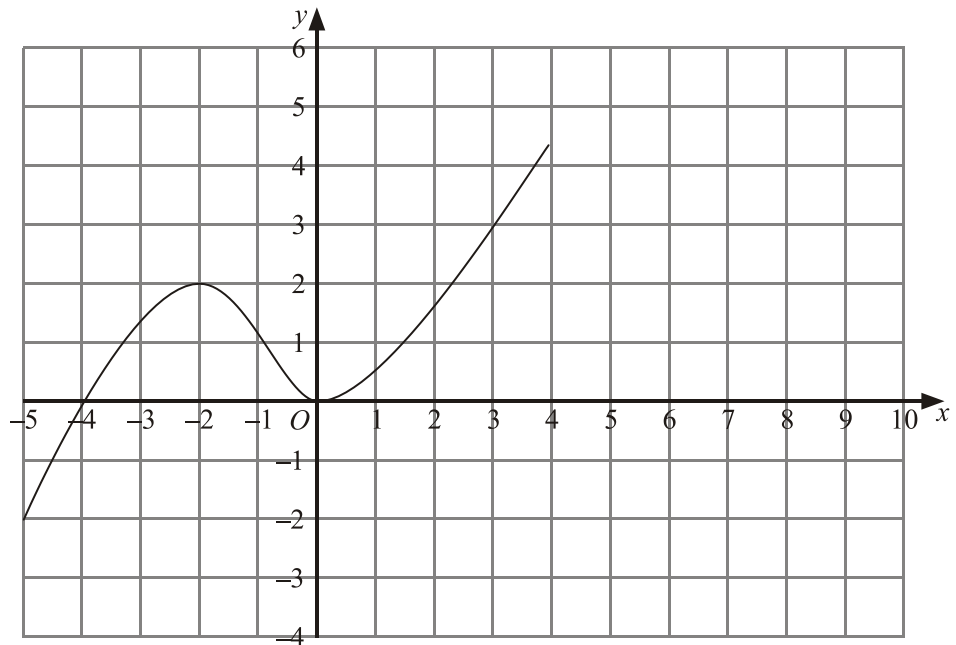
23. The graph of $y = f(x)$ is shown on the grids.

(a) On this grid, sketch the graph of $y = f(x) + 2$



(2)

(b) On this grid, sketch the graph of $y = -f(x)$



(2)

(Total 4 marks)

24. $x^2 - 8x + 23 = (x - p)^2 + q$ for all values of x .

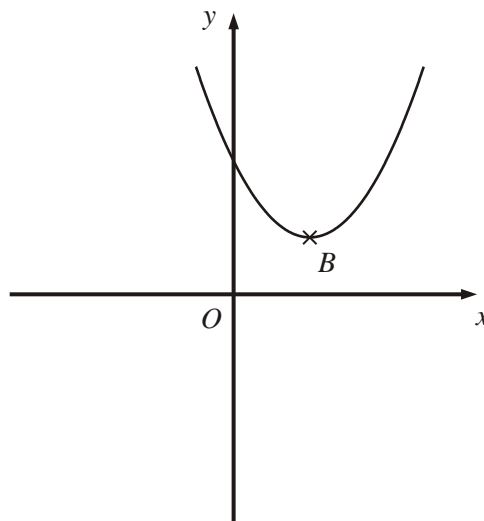
(a) Find the value of p and the value of q .

$p = \dots\dots\dots$

$q = \dots\dots\dots$

(3)

Here is a sketch of the curve with equation $y = x^2 - 8x + 23$



B is the minimum point on the curve.

(b) Find the coordinates of B .

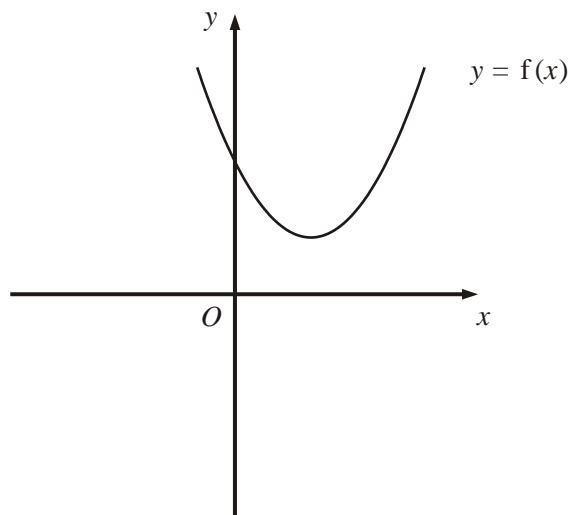
(..... ,)

(1)

The equation of the curve can be written in the form $y = f(x)$,

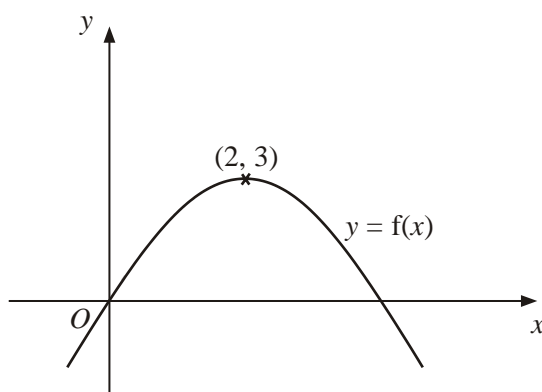
$$\text{where } f(x) = x^2 - 8x + 23$$

(c) On the diagram below, draw a sketch of the curve $y = f(-x)$.



(1)
(Total 5 marks)

25.



The diagram shows part of the curve with equation $y = f(x)$.

The coordinates of the maximum point of this curve are (2, 3).

Write down the coordinates of the maximum point of the curve with equation

(a) $y = f(x - 2)$

(..... ,)

(1)

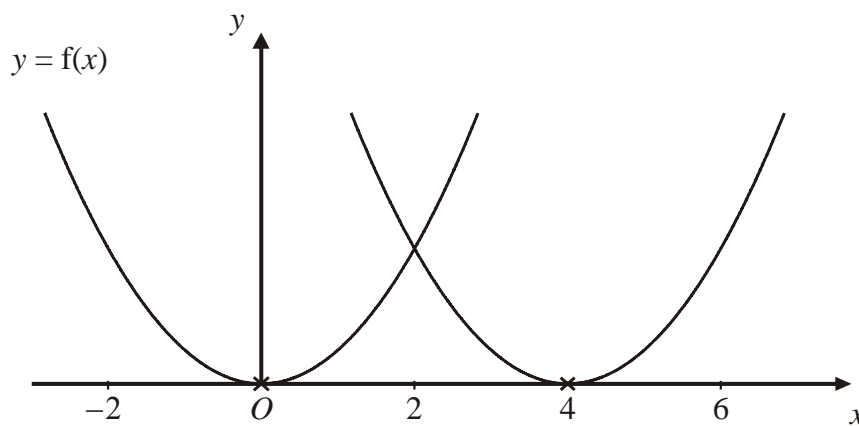
(b) $y = 2f(x)$

(..... ,)

(1)

(Total 2 marks)

26.

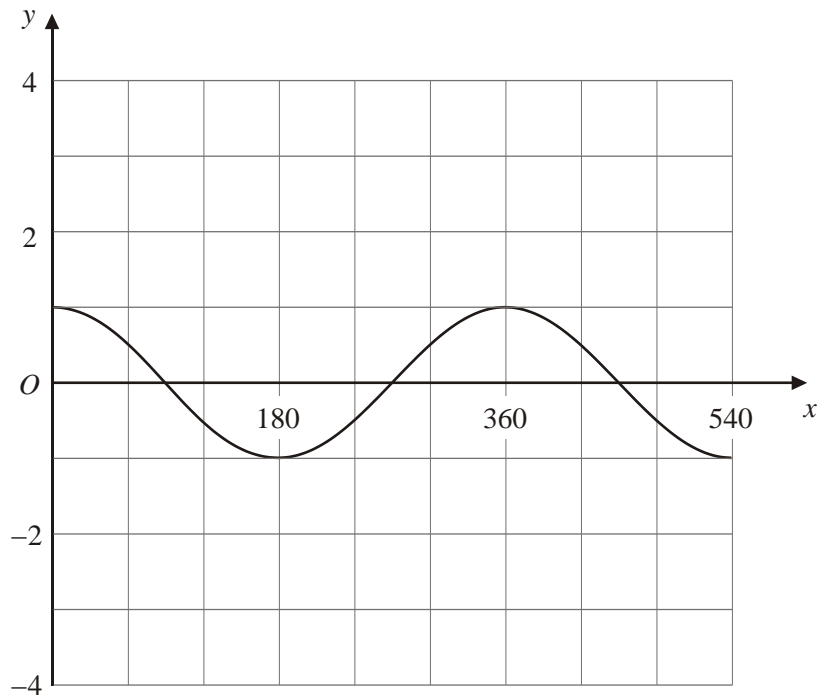


The curve with equation $y = f(x)$ is translated so that the point at $(0, 0)$ is mapped onto the point $(4, 0)$.

(a) Find an equation of the translated curve.

.....

(2)



The grid shows the graph of $y = \cos x^\circ$ for values of x from 0 to 540

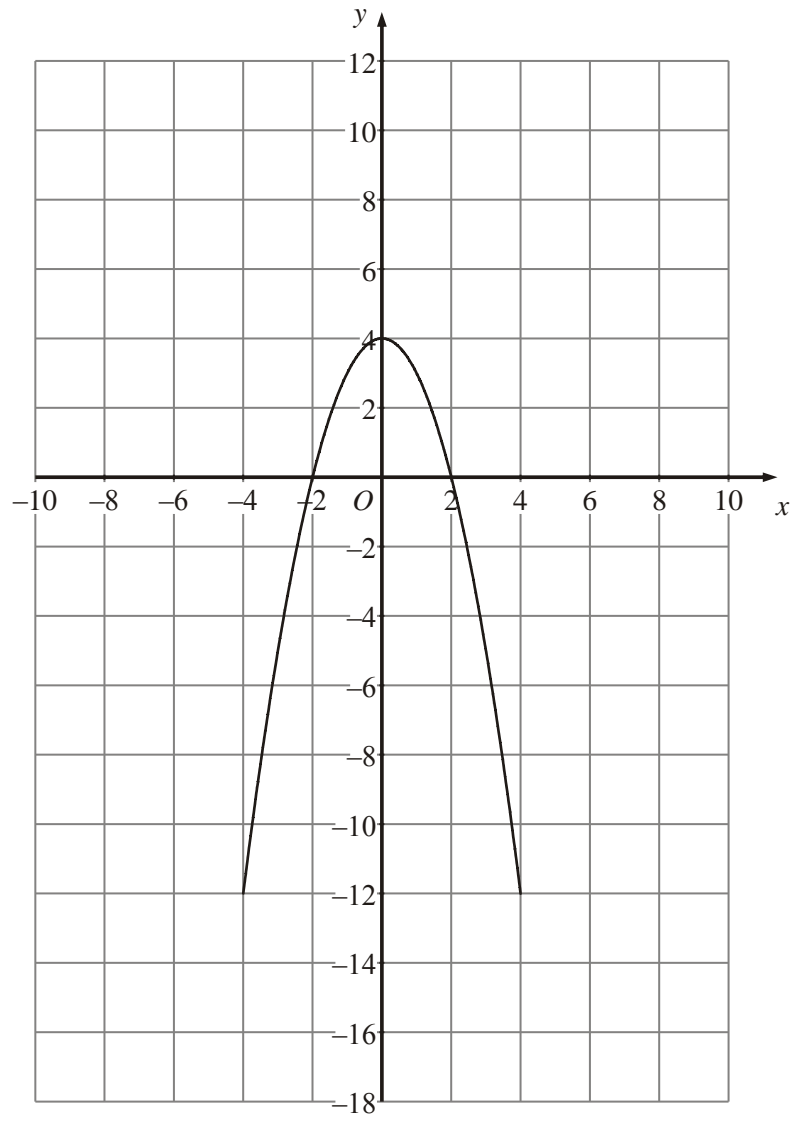
- (b) On the grid, sketch the graph of $y = 3 \cos (2x^\circ)$ for values of x from 0 to 540

(2)

(Total 4 marks)

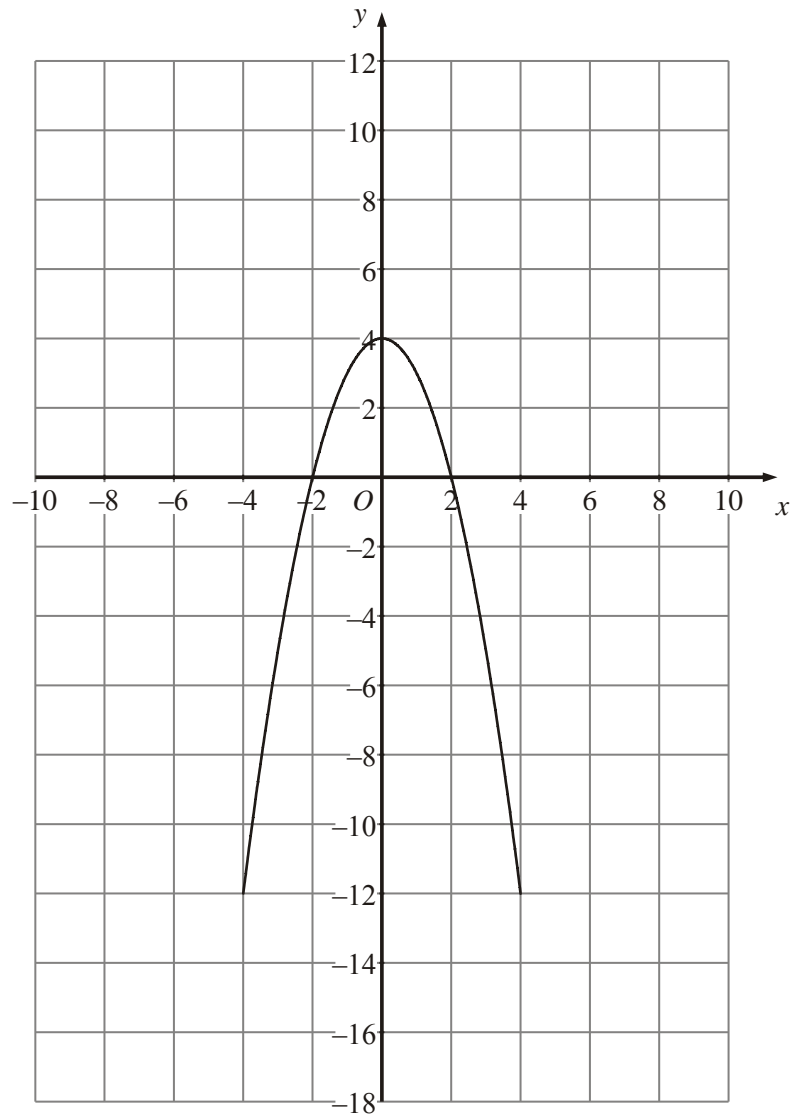
27. The graph of $y = f(x)$ is shown on the grids.

- (a) On this grid, sketch the graph of $y = f(x) - 4$



(2)

(b) On this grid, sketch the graph of $y = f\left(\frac{1}{2}x\right)$.



(2)

(Total 4 marks)

28. For all values of x and m , $x^2 - 2mx = (x - m)^2 - k$

(a) Express k in terms of m .

.....

(2)

The expression $x^2 - 2mx$ has a minimum value as x varies.

- (b) (i) Find the minimum value of $x^2 - 2mx$.
Give your answer in terms of m .

.....

- (ii) State the value of x for which this minimum value occurs.
Give your answer in terms of m .

.....

(3)

(Total 5 marks)

29.

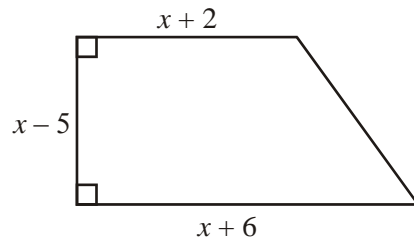


Diagram **NOT**
accurately drawn

The diagram shows a trapezium.

The lengths of three of the sides of the trapezium are $x - 5$, $x + 2$ and $x + 6$.

All measurements are given in centimetres.

The area of the trapezium is 36 cm^2 .

(a) Show that $x^2 - x - 56 = 0$

(4)

(b) (i) Solve the equation $x^2 - x - 56 = 0$

.....

(ii) Hence find the length of the shortest side of the trapezium.

..... cm

(4)

(Total 8 marks)

- 30.** Two numbers have a difference of 15 and a product of 199.75

The larger of the two numbers is x .

- (a) Show that

$$x^2 - 15x - 199.75 = 0$$

(3)

- (b) Solve the equation

$$x^2 - 15x - 199.75 = 0$$

.....
(3)

(Total 6 marks)

- 31.** (a) Factorise $2x^2 - 35x + 98$

.....

- (a) Solve the equation $2x^2 - 35x + 98 = 0$

32.

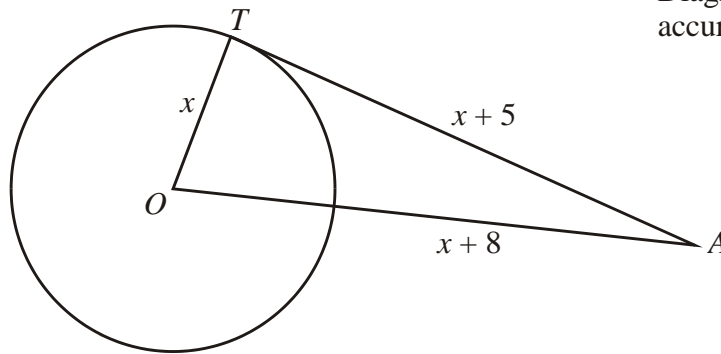


Diagram NOT
accurately drawn

AT is a tangent at T to a circle, centre O .
 $OT = x$ cm, $AT = (x + 5)$ cm, $OA = (x + 8)$ cm.

(a) Show that $x^2 - 6x - 39 = 0$

(4)

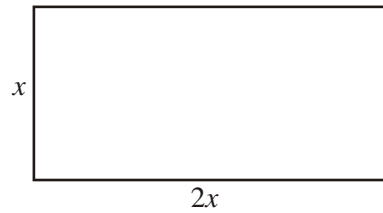
(b) Solve the equation $x^2 - 6x - 39 = 0$ to
find the radius of the circle.
Give your answer correct to 3 significant figures.

..... cm

(3)

(Total 7 marks)

33. The length of a rectangle is twice the width of the rectangle.
The length of a diagonal of the rectangle is 25 cm.



Work out the area of the rectangle.
Give your answer as an integer.

..... cm^2
(Total 3 marks)

34. (a) Solve $x^2 + x + 11 = 14$
Give your solutions correct to 3 significant figures.

..... **(3)**

35. The n th even number is $2n$.

The next even number after $2n$ is $2n + 2$

(a) Explain why.

(1)

(b) Write down an expression, in terms of n , for the next even number after $2n + 2$

.....

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

(3)

(Total 5 marks)

36. Simplify fully

$$\frac{x+3}{4} + \frac{x-5}{3}$$

.....

(Total 3 marks)

37. Simplify

$$\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$$

.....

(Total 3 marks)

38. Simplify fully

$$\frac{2x^2 + 3x + 1}{x^2 - 3x - 4}$$

.....

(Total 3 marks)

39. Write as a single fraction in its simplest form

$$\frac{2x}{x-1} - \frac{7x-3}{x^2-1}$$

.....

(Total 4 marks)

40. Simplify fully

$$\frac{x^2 - 2x - 15}{x^2 - 4x - 21}$$

.....

(Total 3 marks)

41. Simplify fully

$$\frac{x^2 + x - 6}{x^2 - 7x + 10}$$

.....

(Total 3 marks)

42. Simplify fully

$$\frac{x^2 - 8x + 15}{2x^2 - 7x - 15}$$

.....

(Total 3 marks)

43. Simplify

$$\frac{3x^2 - 16x - 35}{9x^2 - 25}$$

.....

(Total 3 marks)

44. Solve the simultaneous equations

$$3x + 4y = 7$$

$$5x - 2y = 16.$$

$$x = \dots\dots\dots$$

$$y = \dots\dots\dots$$

(Total 3 marks)

45. Solve the simultaneous equations

$$3x + 2y = 11$$

$$2x - 5y = 20$$

$$x = \dots\dots\dots$$

$$y = \dots\dots\dots$$

(Total 4 marks)

46. Solve the simultaneous equations.

$$2x + 3y = 0$$

$$x - 3y = 9$$

$$x = \dots\dots\dots, y = \dots\dots\dots$$

(Total 3 marks)

47. Solve the simultaneous equations

$$x^2 + y^2 = 5$$

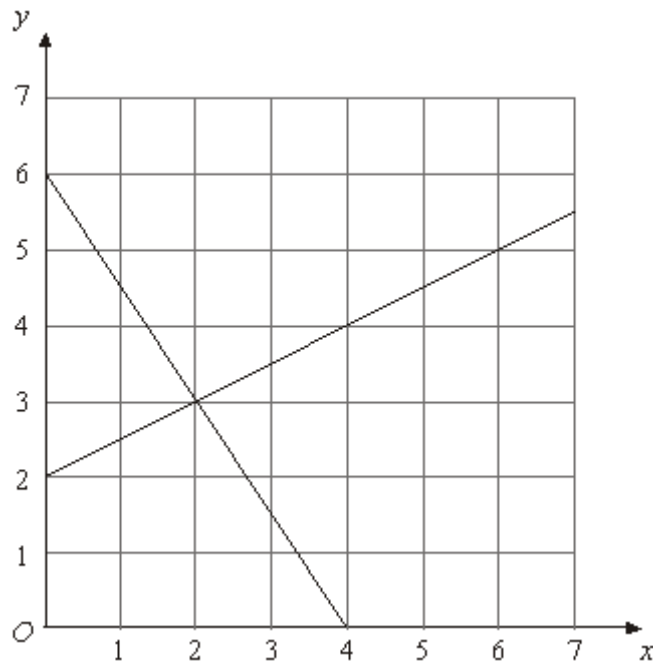
$$y = 3x + 1$$

$$x = \dots\dots\dots y = \dots\dots\dots$$

$$\text{or } x = \dots\dots\dots y = \dots\dots\dots$$

(Total 6 marks)

48.



The diagram shows graphs of $y = \frac{1}{2}x + 2$ and $2y + 3x = 12$

(a) Use the diagram to solve the simultaneous equations

$$y = \frac{1}{2}x + 2$$
$$2y + 3x = 12$$

$$x = \dots\dots\dots y = \dots\dots\dots$$

(1)

(b) Find an equation of the straight line which is parallel to the

line $y = \frac{1}{2}x + 2$ and passes through the point (0, 4).